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Performance Analysis of Markovian Queueing Model in presence of Encouraged Arrivals

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Abstract

Progress of business has been estimated by number of customers associated with the company. So, to increase the business of the company, customer's encouragement is required by applying some reward, gift voucher or additional services and benefits etc. In this paper, we develop a Finite capacity and single server Markovian queueing model with encouraged arrivals. Here we obtain steady state solution of the model by recursive procedure and some performance measures are discussed.

Keywords*: Markov process, Encouraged arrivals, steady state solution, system capacity, queueing system.*

1. Introduction

Queueing models are effectively used in the design and analysis of service system, telecommunication system, traffic systems and many more. Queueing theory is a Mathematical study of waiting lines or queues. The formation of a queue is a common phenomenon which occurs whenever the current demand for service exceeds the current capacity to provide that service. Queues may be seen at railway station, cinema ticket windows, billing counters of super market etc. The main objective of queueing theory is to minimize the congestion and provide best level of service at minimum possible cost. The waiting lines arise from congestion which occurs from time to time as a result of irregularities in arrival of customers or in the length of time needed to service the customers. Assuming that the irregularities mentioned above follow some probabilistic laws, the queueing theory attempts to study statistical fluctuations in these irregularities. It also helps to determine the probability of number of customers in the system, the average system size, the average queue length, average waiting time of customers in the system, average server utilization etc.

Now a day's managing business has become a highly challenging task due to highly uncertain business and economic environment in industries in present era. One of the most uncertain aspects of business is customer's behavior. Customers have become more selective in present time. A customer may get impatient due to higher level of expectations, delay in service, lack of facility otherwise he decides to leave the service facility before completion of service. This type of behavior leads to the loss in profit and goodwill of the company and it becomes the most important threat to any business industries. But, when it comes to sensitive businesses like investment, selecting a restaurant for dinner, selecting a service center, choosing a saloon etc. more number of customers with a particular firm becomes the attracting factor for other customers. Level of impatience of customers depends upon the amount of trust they show with particular product or company. Customers are willing to spend more time with high level of patience with the firms having a large consumer's base. For instance, if someone is planning to dine out, he is willing to wait for much longer in order to get access to a well-known restaurant. It is also obvious that well-known brands have a large customers' base. Hence, a large customers' base also works as a trust factor for a customer and the patience level of the customer is high in such type of cases. This behavior is referred to as reverse reneging, according to which, higher system size results in high patience and vice-versa. By keeping this in mind, researchers across the world study various stochastic queuing models with reneging. In this model, reneging is a function of system size. When there are more number of customers in the system and vice versa, which is reneging in reverse sense called as Reverse Reneging. Barrer [1] obtained the steady - state solution of a single channel queuing model having poisson input, exponential holding time, random selection where impatient customers leave the service facility after a wait of certain time. Hunt [2] studied the problems of serial queues in the steady state with Poisson assumptions. In these studies it is assumed that the unit must go through each service channel without leaving the system. The busy period for the Poisson queue is studied by Takcacs [3]. Some queuing problems with balking and reneging was studied by Ancker and Gafarian [4]. Ancker and Gafarian [5] obtain results for a pure balking system by setting the reneging parameter equal to zero. The premier work on customer impatience in queuing theory appears in Haight [6]. He investigated M/M/1 queue with balking in which there is a greatest queue length at which the arrival will not balk. Haight [7] studies a queue with reneging in which he studies the problem like how to make rational decision while waiting in the queue, the probable effect of this decision etc. Kumar [8] discussed the busy period analysis of an M/M/1 queue with balking. Armony et al. [9] study sensitivity of the optimal capacity to customer impatience. They observe that the prevention of reneging during service can substantially reduce the total cost of lost sales and capacity. Choudhury and Medhi [10] presented some aspects of balking and reneging in finite buffer queue. Wang et al. [11] performed reliability analysis using Laplace transform techniques. Recently, Rakesh and Sumeet [12] studied a queuing model with retention of reneged customers and balking. Kumar & Sharma [13] developed queueing model with reneging, balking and retention of reneged customers. They also developed some performance measures and analyzed the results numerically. Awasthi [14] analyzed performance of M/M/1/K finite capacity model with reverse balking and reverse reneging. He developed steady solution of the model and some performance measures of the model also described. Awasthi & Sharma [15] studied Markovian queueing model with encouraged arrivals and reneging of customers. They also derived some performance measure for the developed queueing model.

In this paper, we present steady-state analysis of the stochastic model and derive some important measures of performance for the queueing model. Rest of the paper is arranged as follows: Basic queueing model has been described with some other related terms in section 2. In section 3, the assumptions of the queueing model are described. In section 4, Mathematical Analysis of the queueing model has been done. Steady state solution of the model has been developed in section 5. In section 6, Performance measures for Queueing model have been derived. Conclusion of the research paper is described in section 7.

2. The Basic Queueing Model

The basic queueing model consisits of three main systems, the arriving objects, a queue of arrived objects wating to be processed by a processing unit, and the processing unit.

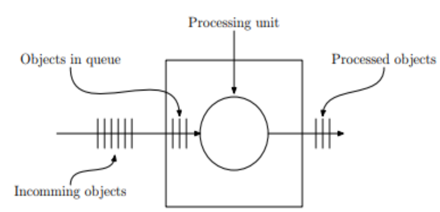

Figure 1: Basic Diagram for queueing model

In many systems we don’t have exact knowledge about the arrival process of objects, and we don’t have exact knowledge of the time the processing unit takes to process each object. So, the basic model given in fig. 1 can be seen as the interaction between two stochastic systems, the arrival process and the processing of objects. In queueing system, customer arrives and waits for service, after getting service they leave the system.

2.1 The Kendall Notation

This section describes the standard notation for queueing models. The full Kendall naming convention for queue models is

A/S/c/b/k/P

In the Kendall naming convention, the different symbols have the following meaning:

In the Kendall naming convention, the different symbols have the following meaning:

A: The probability distribution for the arrival process

S: The probability distribution for the service process

C: The number of parallel service channels available in the service system

b: System capacity restriction or capacity of the system or the maximum number of customers allowed in the queueing system (either being served or waiting for service)

k: Maximum potential population size

P: Policy for queueing discipline

2.2 Markov process

Markov process is a stochastic or random process, that is used in decision problems in which the probability of transition to any future state depends on the current state and not on the manner in which the specific state was reached. Mathematically,

$$P\{X_n \mid X_{n-1}, X_{n-2,} ..., X_2, X_1\} = P\{X_n \mid X_{n-1}\}$$

Markov analysis involves the studying of the present behavior of a system to predict the future behavior of the same system. That is introduced by Russian mathematician, Andrey A Markov. General theory concerning Markov process was developed by A N Kolmogorov, W. Feller and others. Markov processes are a certain special class of mathematical models that are used in decision problems associated with dynamic systems. Markov processes are widely used, perhaps, as marketing aid for examining and predicting the customer behavior concerning their loyalty to one brand of a given product and their switching patterns to other brands of the same product.

2.3 Poisson distribution

The Poisson distribution is used to determine the probability of a certain number of arrivals occurring in a given time period. The Poisson distribution with parameter λ is given by

$$\frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

Where n is the number of arrivals. We find that if we set n=0, the Poisson distribution gives us $e^{-\lambda t}$. Which is equal to $P(T \succ t)$ from the exponential distribution. The interarrival time here, of course, is the time between customer arrivals, and thus is a period of time with zero arrival.

2.4 Exponential Distribution

The exponential distribution is one of the widely used continuous distributions. It is often used to model the time elapsed between events. A continuous random variable X is said to have an exponential distribution with parameter $\lambda \succ 0$, shown as $X - Exponentid\,(\lambda)$, if its PDF is given by

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \succ 0 \\ 0, & otherwise \end{cases}$$

If T is a random variable that represents interarrival times with the exponential distribution, then

$$P(T \le t) = 1 - e^{-\lambda t} and\; P(T \succ t) = e^{-\lambda t}$$

This distribution lends itself well to modeling customer interarrival times or service times for a number of reasons. The one is the fact that the exponential function is a strictly decreasing function of t. This means that after an arrival has occurred, the amount of waiting time until the next arrival is more likely to be small than large. Another important property of the exponential distribution is what is known as the no-memory property. The no memory property suggests that the time until the next arrival will never depend on how much time has already passed. This makes intuitive sense for a model where we are measuring customer arrivals because the customers actions are clearly independent of one another.

3. Assumptions and formulation of the Queueing model

A single server Markovian queueing model is formulated with the following assumptions:

1) Customers arrive in the queueing system one by one according to a Poisson distribution with mean arrival rate $\lambda(1+e)$, where e denotes the percentage change in number of customers after launching some promotional schemes. The inter-arrival times are independently, identically and exponentially distributed with parameter $\lambda(1+e)$.
2) There is single server in the service facility of the queueing system. The service times are independently, identically and exponentially distributed with parameter μ .
3) The capacity of the queueing system is K.
4) Customers are served in order to their arrival i.e. the Service discipline of the queueing system is First Come, First served.
5) $P_k(t)$ be the probability that there is k customer in the system at time t.
6) $P_0(t)$ be the probability that there is no customer in the system at time t.

4. Mathematical Analysis of the queueing model

The transition rate diagram for the queueing model is described in the figue 2.

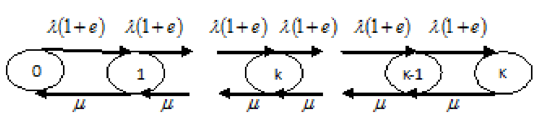

Figure 2: Transition rate diagram for the model

The governing differential difference equations of the model are given by

$$\frac{dP_0(t)}{dt} = -\lambda(1+e)P_0(t) + \mu\ P_1(t); \quad for\, k = 0 \qquad (1)$$

$$\frac{dP_k(t)}{dt} = \lambda(1+e)P_{k-1}(t) - [\lambda(1+e) + \mu]P_k(t)$$

$$+\mu P_{k+1}(t); \quad 1 \le k \prec K-1 \qquad (2)$$

$$\frac{dP_K(t)}{dt} = \lambda(1+e)P_{K-1}(t) - \mu P_K(t); \quad for\, k = K \qquad (3)$$

5. Steady State Solutions of the queueing system

In the steady state position, $\lim_{k\to\infty} P_k(t) = P_k$.

Therefore

$$0 = -\lambda(1+e)P_0(t) + \mu\ P_1(t); \quad for\, k = 0 \qquad (4)$$

$$0 = \lambda(1+e)P_{k-1}(t) - [\lambda(1+e) + \mu]P_k(t)$$

$$+\mu P_{k+1}(t); \quad 1 \le k \prec K-1 \qquad (5)$$

$$0 = \lambda(1+e)P_{K-1}(t) - \mu P_K(t); \quad for\, k = K \qquad (6)$$

Solving equations (6)-(10) using iterative procedure, the steady state solution of the model is given by

$$P_k = \Pr\{k\ customers\ in\ the\ system\}$$

$$= \left[\frac{\lambda(1+e)}{\mu}\right]^k P_0,\ 1 \le k \le K-1 \qquad (7)$$

And the probability that the system is full is given by

$$P_K = \Pr\{system\ is\ \ full\} = \left[\frac{\lambda(1+e)}{\mu}\right]^K P_0,\ \ k = K \qquad (8)$$

Using normalization condition $\sum_{i=0}^{K} P_k = 1$

$$or\ P_0 + \sum_{k=1}^{K} P_k = 1$$

$$Or\ P_0 = \frac{1}{\left[1 + \sum_{k=1}^{K}\left[\frac{\lambda(1+e)}{\mu}\right]^k\right]} \qquad (9)$$

6. Performance Measures of the queuing system

Here we derive some important measures of performance for the developed queueing model.

6.1 Average number of customers in a system (Expected Size of the System)

Expected number of customers in the system is known as system size. The expected system size is given by

$$L_s = \sum_{i=1}^{K} kP_k$$

$$= \sum_{k=1}^{K} k\left[\frac{\lambda(1+e)}{\mu}\right]^k P_0 \qquad (10)$$

6.2 Expected queue length in the system

The expected queue length in the system is given by

$$L_q = \sum_{k=1}^{K} (k-1)P_k$$

$$= \sum_{k=1}^{K} (k-1)\left[\frac{\lambda(1+e)}{\mu}\right]^k P_0 \qquad (11)$$

7. Conclusion

In this paper, a multi-server Markovian queueing model with encouraged arrival of customers' is developed. Steady state solution of the model is derived by using iterative procedure. Some important performance measures have been derived for the stochastic model developed. This analysis can be used in industries to grow the business potential and retain loyal customers. In future cost analysis of the model can be presented with optimization. This model is limited to finite capacity. The infinite capacity model can also be developed.

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